

Integrating Loop Acceleration into Bounded Model Checking

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Safety Problems by Example

Pre Variables: x, y Post Variables: x', y' (domain: \mathbb{Z} – in this talk)Start States: $\psi_s := x \leq 0 \wedge y \leq 0$ Error States: $\psi_e := y \geq 100$

Transition Formula: $\tau := \underbrace{(x < 100 \wedge x' = x + 1 \wedge y' = y)}_{\tau_l} \vee \underbrace{(x = 100 \wedge x' = 0 \wedge y' = y + 1)}_{\tau_r}$ (QF)

Example (Transition Relation)

 $(23, 42) \rightarrow_{\tau} (24, 42)$ as $[x/23, y/42, x'/24, y'/42] \models \tau$

Dependency Graphs



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BMC

 $b \leftarrow 0$; add($vr_b(\psi_s)$)where $vr_b(x) := x_b, vr_b(x') := x_{b+1}, \dots$ **while** \top **do**| push(); add($vr_b(\psi_e)$)| **if** check() = sat **then** return trace() **else** pop(); add($vr_b(\tau)$)| **if** check() = unsat **then** return safe **else** $b++$

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- ...
- $vr_0(\psi_s) \wedge vr_0(\tau) \wedge \dots \wedge vr_{10099}(\tau) \wedge vr_{10100}(\psi_e) \leadsto \text{sat}$
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Sloooow, basically brute force!

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Definition (Acceleration)

A function with $\text{accel}(\tau) := \tau^{\oplus}$ where $\rightarrow_{\tau^{\oplus}} \subseteq \rightarrow_{\tau}^+$.

Example

How? Many techniques, e.g...

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How? Many techniques, e.g...

• monotonicity criteria: $n \Rightarrow x < x'$

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- monotonicity criteria: $\tau_i \implies x < x'$
- recurrence solving

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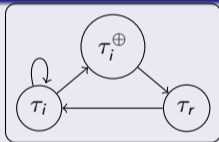
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ABMC

```

b ← 0;  add(vr_b(ψ_s))
if check() = unsat then return safe
while ⊤ do
  push();  add(vr_b(ψ_e))
  if check() = sat then return τ̄
  else
    pop();  τ̄ ← trace();
    if τ̄ ends with loop τ̄o then
      | add(vr_b(τ ∨ accel(τ̄o)))
    else add(vr_b(τ))
  if check() = unsat then return safe else b++
  
```



b	$\vec{\tau}$	SMT Problem	Model
0	\emptyset	$vr_0(\psi_s) \wedge vr_0(\tau)$	$[x_1/1, \dots / 0]$
1	$[\tau]$	$\dots \wedge vr_1(\tau \vee \tau_i^\oplus)$	$\dots \cup [x_2/100, y_2/0]$
2	$[\tau, \tau_i^\oplus]$	$\dots \wedge vr_2(\tau)$	$\dots \cup [x_3/0, y_3/1]$
3	$[\tau, \tau_i^\oplus, \tau_r]$	$\dots \wedge vr_3(\tau \vee \text{accel}(\vec{\tau}^o))$	$\dots \cup [x_4/0, y_4/100]$
4		$\dots \wedge vr_4(\psi_e)$	

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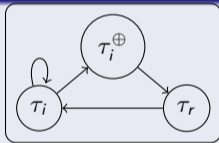
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$\tau_i^\oplus := x + n \leq 100 \wedge x' = x + n \wedge y' = y \wedge n > 0$

ABMC

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b ← 0; add(vrb(ψs))
if check() = unsat then return safe
while ⊤ do
  push(); add(vrb(ψe))
  if check() = sat then return  $\vec{\tau}$ 
  else
    pop();  $\vec{\tau} \leftarrow \text{trace}()$ ;
    if  $\vec{\tau}$  ends with loop  $\vec{\tau}^\circ$  then
      | add(vrb( $\tau \vee \text{accel}(\vec{\tau}^\circ)$ ))
      | else add(vrb( $\tau$ ))
  if check() = unsat then return safe else b++
  
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b	$\vec{\tau}$	SMT Problem	Model
0	\emptyset	$\text{vr}_0(\psi_s) \wedge \text{vr}_0(\tau)$	$[x_1/1, \dots / 0]$
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2	$[\tau, \tau_i^\oplus]$	$\dots \wedge \text{vr}_2(\tau)$	$\dots \cup [x_3/0, y_3/1]$
3	$[\tau, \tau_i^\oplus, \tau_r]$	$\dots \wedge \text{vr}_3(\tau \vee \text{accel}(\vec{\tau}^\circ))$	$\dots \cup [x_4/0, y_4/100]$
4		$\dots \wedge \text{vr}_4(\psi_e)$	

Leading Example

Start States: $\psi_s := x \leq 0 \wedge y \leq 0$

Error States: $\psi_e := y \geq 100$

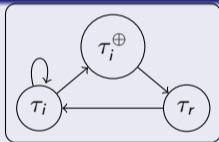
Transition Formula: $\tau := \underbrace{(x < 100 \wedge x' = x + 1 \wedge y' = y)}_{\tau_i} \vee \underbrace{(x = 100 \wedge x' = 0 \wedge y' = y + 1)}_{\tau_r}$

$\tau_i^\oplus := x + n \leq 100 \wedge x' = x + n \wedge y' = y \wedge n > 0$

ABMC

```

b ← 0; add(vr_b(ψ_s))
if check() = unsat then return safe
while ⊤ do
  push(); add(vr_b(ψ_e))
  if check() = sat then return τ̄
  else
    pop(); τ̄ ← trace();
    if τ̄ ends with loop τ̄o then
      | add(vr_b(τ ∨ accel(τ̄o)))
    else add(vr_b(τ))
  if check() = unsat then return safe else b++
  
```



b	$\vec{\tau}$	SMT Problem	Model
0	\square	$vr_0(\psi_s) \wedge vr_0(\tau)$	$[x_1/1, \dots / 0]$
1	$[\tau]$	$\dots \wedge vr_1(\tau \vee \tau_i^\oplus)$	$\dots \cup [x_2/100, y_2/0]$
2	$[\tau, \tau_i^\oplus]$	$\dots \wedge vr_2(\tau)$	$\dots \cup [x_3/0, y_3/1]$
3	$[\tau, \tau_i^\oplus, \tau_r]$	$\dots \wedge vr_3(\tau \vee \text{accel}(\vec{\tau}^o))$	$\dots \cup [x_4/0, y_4/100]$
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Leading Example

Start States: $\psi_s := x \leq 0 \wedge y \leq 0$

Error States: $\psi_e := y \geq 100$

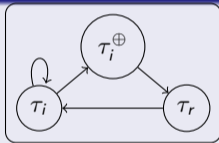
Transition Formula: $\tau := \underbrace{(x < 100 \wedge x' = x + 1 \wedge y' = y)}_{\tau_i} \vee \underbrace{(x = 100 \wedge x' = 0 \wedge y' = y + 1)}_{\tau_r}$

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ABMC

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b ← 0;  add(vr_b(ψ_s))
if check() = unsat then return safe
while ⊤ do
  push();  add(vr_b(ψ_e))
  if check() = sat then return τ̄
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    pop();  τ̄ ← trace();
    if τ̄ ends with loop τ̄o then
      | add(vr_b(τ ∨ accel(τ̄o)))
    else add(vr_b(τ))
  if check() = unsat then return safe else b++
  
```



b	$\vec{\tau}$	SMT Problem	Model
0	\emptyset	$vr_0(\psi_s) \wedge vr_0(\tau)$	$[x_1/1, \dots / 0]$
1	$[\tau]$	$\dots \wedge vr_1(\tau \vee \tau_i^\oplus)$	$\dots \cup [x_2/100, y_2/0]$
2	$[\tau, \tau_i^\oplus]$	$\dots \wedge vr_2(\tau)$	$\dots \cup [x_3/0, y_3/1]$
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Leading Example

Start States: $\psi_s := x \leq 0 \wedge y \leq 0$

Error States: $\psi_e := y \geq 100$

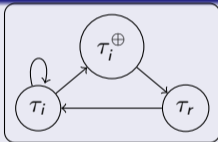
Transition Formula: $\tau := \underbrace{(x < 100 \wedge x' = x + 1 \wedge y' = y)}_{\tau_i} \vee \underbrace{(x = 100 \wedge x' = 0 \wedge y' = y + 1)}_{\tau_r}$

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ABMC

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b ← 0;  add(vr_b(ψ_s))
if check() = unsat then return safe
while ⊤ do
  push();  add(vr_b(ψ_e))
  if check() = sat then return  $\vec{\tau}$ 
  else
    pop();   $\vec{\tau} \leftarrow \text{trace}()$ ;
    if  $\vec{\tau}$  ends with loop  $\vec{\tau}^\circ$  then
      | add(vr_b( $\tau \vee \text{accel}(\vec{\tau}^\circ)$ ))
      else add(vr_b( $\tau$ ))
  if check() = unsat then return safe else b++
  
```



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0	\square	$\text{vr}_0(\psi_s) \wedge \text{vr}_0(\tau)$	$[x_1/1, \dots / 0]$
1	$[\tau]$	$\dots \wedge \text{vr}_1(\tau \vee \tau_i^\oplus)$	$\dots \cup [x_2/100, y_2/0]$
2	$[\tau, \tau_i^\oplus]$	$\dots \wedge \text{vr}_2(\tau)$	$\dots \cup [x_3/0, y_3/1]$
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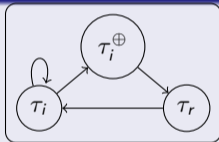
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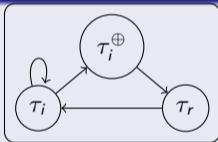
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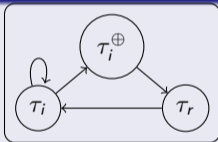
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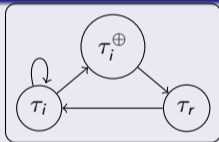
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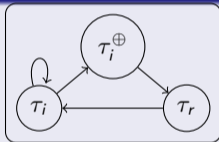
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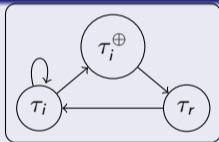
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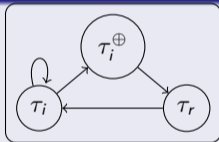
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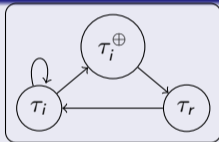
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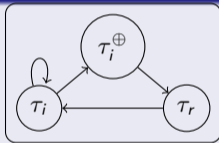
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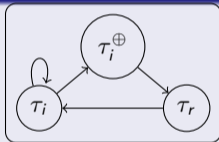
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Start States: $\psi_s := x \leq 0 \wedge y \leq 0$

Error States: $\psi_e := y \geq 100$

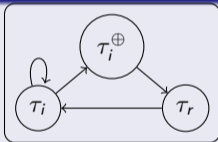
Transition Formula: $\tau := \underbrace{(x < 100 \wedge x' = x + 1 \wedge y' = y)}_{\tau_i} \vee \underbrace{(x = 100 \wedge x' = 0 \wedge y' = y + 1)}_{\tau_r}$

$\tau_i^\oplus := x + n \leq 100 \wedge x' = x + n \wedge y' = y \wedge n > 0$

ABMC

```

b ← 0;  add(vr_b(ψ_s))
if check() = unsat then return safe
while ⊤ do
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  if check() = sat then return τ̄
  else
    pop();  τ̄ ← trace();
    if τ̄ ends with loop τ̄o then
      | add(vr_b(τ ∨ accel(τ̄o)))
    else add(vr_b(τ))
  if check() = unsat then return safe else b++
  
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b	$\vec{\tau}$	SMT Problem	Model
0	\square	$vr_0(\psi_s) \wedge vr_0(\tau)$	$[x_1/1, \dots / 0]$
1	$[\tau_i]$	$\dots \wedge vr_1(\tau \vee \tau_i^\oplus)$	$\dots \cup [x_2/100, y_2/0]$
2	$[\tau_i, \tau_i^\oplus]$	$\dots \wedge vr_2(\tau)$	$\dots \cup [x_3/0, y_3/1]$
3	$[\tau_i, \tau_i^\oplus, \tau_r]$	$\dots \wedge vr_3(\tau \vee \text{accel}(\vec{\tau}^o))$	$\dots \cup [x_4/0, y_4/100]$
4		$\dots \wedge vr_4(\psi_e)$	

Leading Example

Start States: $\psi_s := x \leq 0 \wedge y \leq 0$

Error States: $\psi_e := y \geq 100$

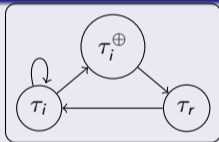
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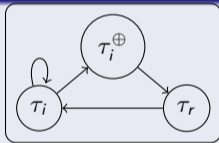
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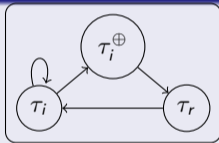
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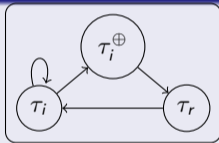
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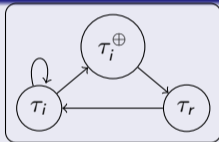
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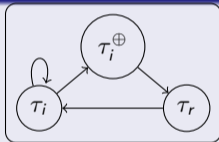
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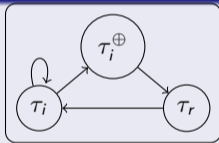
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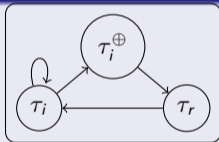
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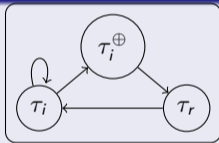
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  else
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    if  $\vec{\tau}$  ends with loop  $\vec{\tau}^\circ$  then
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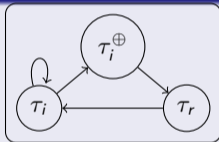
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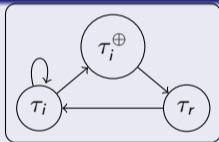
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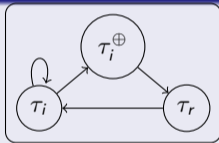
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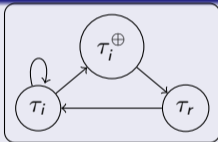
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Leading Example

Start States: $\psi_s := x \leq 0 \wedge y \leq 0$

Error States: $\psi_e := y \geq 100$

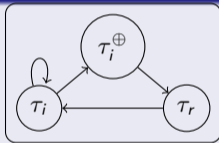
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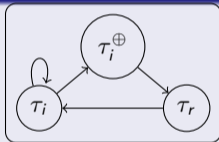
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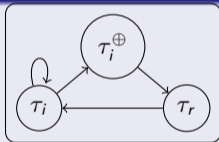
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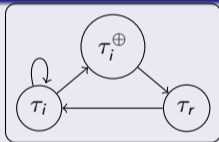
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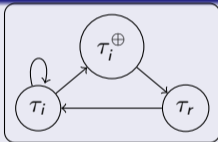
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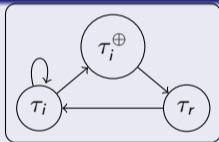
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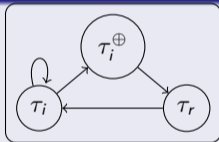
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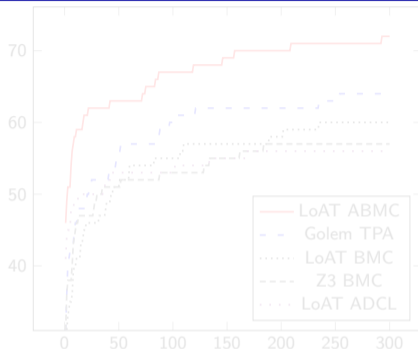
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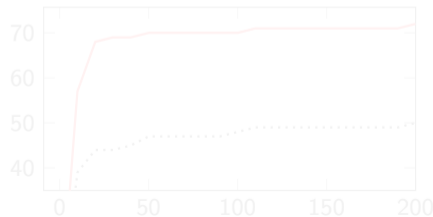


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CHC-Comp '23 Benchmarks	unsafe ✓	safe ✓
LoAT ABMC	72	75
Golem TPA	64	83
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Z3 BMC	57	21
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Spacer (Z3)	51	151
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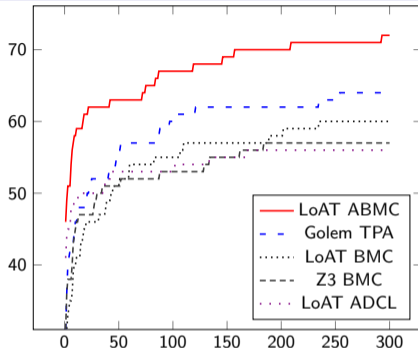


x: runtime in s
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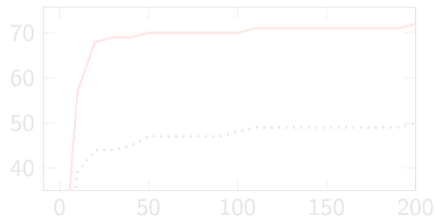


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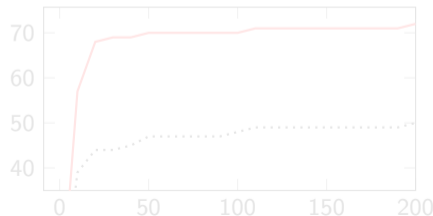
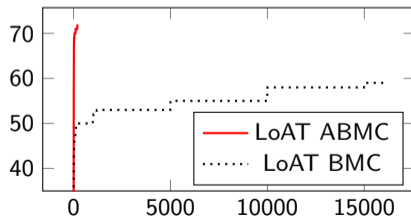
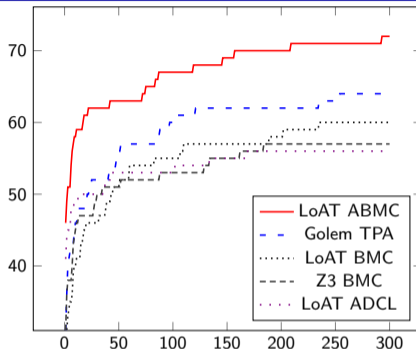


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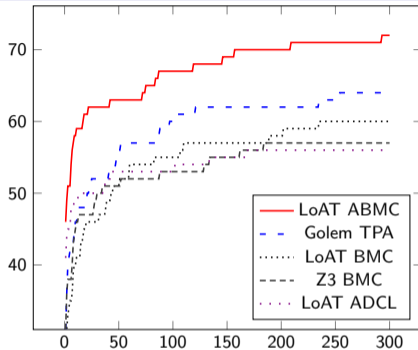


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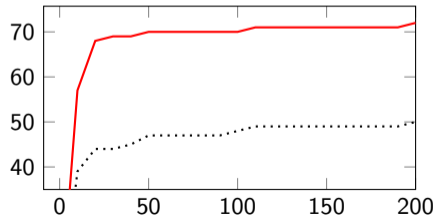
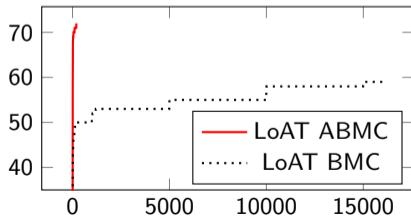
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See paper for...

- lazy exploration of dependency graph
- heuristics to fine-tune acceleration
- proving safety via **Blocking Clauses**

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